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THERMOELASTOPLASTIC CREEP STRESS ANALYSIS FOR A THICK-WALLED TUBE

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Abstract—This paper presents a theoretical study of the stresses in an infinite thick-walled tube subjected to rapid inner-surface heating. Quasi-static, uncoupled, thermoelastoplastic creep analysis based on the incremental theory of plasticity and the Mises–Mises type of creep theory is formulated for a method of successive elastic solutions. The material of the cylinder is assumed to have temperature dependent properties and to be characterized by the Ramberg–Osgood's stress strain relation and Norton's law for secondary creep.

1. INTRODUCTION

The actual properties of a material at elevated temperature can not be described precisely without taking into consideration the following phenomena[1]: (1) The temperature dependence of physical coefficients; (2) the decrease in magnitude of the material yield stress with rising temperature; (3) time-dependent strain and stress response of the material with regard to variations of the temperature field. Recently with attention to the phenomena of the first and second category just referred to, the transient thermal stresses for a hollow sphere[2] and for an infinite circular solid cylinder[3] of the Ramberg–Osgood type material[4] with temperature-dependent properties have been solved by use of the incremental strain theory of plasticity. At sufficiently high temperatures creep deformation, just as in time-independent plasticity, takes place under a constant, nonhydrostatic state of stress. The situation becomes exaggerated when either the temperature or the stress level is raised.

Though the phenomena of the third category ought to be taken into account sufficiently provided that the physical primary creep could be obtained at the high stress level in short time, this paper treats the thermoelastoplastic creep deformation during the transient state of temperature and relaxation of thermal stress at the steady state in the thick-walled tube after a sudden temperature rise on its inner surface, considering only the statical primary creep with Norton's law. With the physical coefficients[5] characterizing the mechanical and thermal behaviour of the carbon steel (0.40 Mn, 0.25 Si, 0.12 C), the method of successive elastic solutions[6] is used for the numerical calculation.

2. THEORETICAL ANALYSIS

2.1 *Fundamental equations for stresses and strains*

We consider an infinite thick-walled tube of inner and outer radius a and b, which is initially stress free. It is subjected to an axially symmetric temperature distribution that varies with time. We assume that the initial uniform temperature of the cylinder is equal to zero and that body forces and surface tractions are absent. In the following analysis the problem is treated in a quasi-static sense, and the inertia terms are neglected.

If the temperature at a generic radius r is T at time t, then the total strain is the sum of an elastic component of strains, a thermal part, a plastic component, and a creep component. Then, we have

$$\epsilon^{*t}_{ij} = \epsilon^{*e}_{ij} + \delta_{ij}(1-\nu)\int \alpha^* \, d\theta + \epsilon^{*p}_{ij} + \epsilon^{*c}_{ij} \tag{1}$$

where the subscripts i, j denote components of the appropriate tensor and $\epsilon^*_{ij} = (1-\nu)\epsilon_{ij}/\alpha_0 T_0, \ldots$. Moreover, we have employed the dimensionless variables $\bar{\theta} = T/T_0$ where T_0 denotes any conveniently chosen reference temperature and is taken in this case to be equal to the constant surface temperature to which the boundary of the thick-walled tube is suddenly exposed. Therefore, we can derive the fundamental equations for stresses and strains similarly as in [3] by substituting $\epsilon_r^{*p}+\epsilon_r^{*c}$, $\epsilon_\theta^{*p}+\epsilon_\theta^{*c}$ and $\epsilon_z^{*p}+\epsilon_z^{*c}$ for ϵ_r^{*p}, ϵ_θ^{*p} and ϵ_z^{*p}. The following stress components for an axisymmetric problem can be expressed similarly as in [3].

$$\begin{aligned}
\sigma_r^* &= -\frac{1-\nu}{1-2\nu}E^*\int\alpha^*\,d\bar\theta+\frac{E^*}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_r^*+\nu(\epsilon_\theta^*+\epsilon_z^*)]-\frac{E^*}{1+\nu}(\epsilon_r^{*p}+\epsilon_r^{*c}),\\
\sigma_\theta^* &= -\frac{1-\nu}{1-2\nu}E^*\int\alpha^*\,d\bar\theta+\frac{E^*}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_\theta^*+\nu(\epsilon_z^*+\epsilon_r^*)]-\frac{E^*}{1+\nu}(\epsilon_\theta^{*p}+\epsilon_\theta^{*c}),\\
\sigma_z^* &= -\frac{1-\nu}{1-2\nu}E^*\int\alpha^*\,d\bar\theta+\frac{E^*}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_z^*+\nu(\epsilon_r^*+\epsilon_\theta^*)]-\frac{E^*}{1+\nu}(\epsilon_z^{*p}+\epsilon_z^{*c})
\end{aligned}\tag{2}$$

where, $\sigma_{ij}^*=(1-\nu)\sigma_{ij}/E_0\alpha_0T_0$. The coefficient of thermal expansion α, the conductivity K, the elastic modulus E, the yield stress σ_1, the mass density γ, and the specific heat C have each been defined with two factors as in the previous work[2].

$$\alpha=\alpha_0\alpha^*(\bar\theta),\ K=K_0K^*(\bar\theta),\ E=E_0E^*(\bar\theta),\ \sigma_1=\sigma_{10}\sigma^*(\bar\theta),\ \gamma=\gamma_0\gamma^*(\bar\theta),\ C=C_0C^*(\bar\theta)\tag{3}$$

where α^*, E^*, K^* and σ_1^* are defined by

$$\alpha^*=1+\alpha_1\bar\theta,\ K^*=1-K_1\bar\theta,\ E^*=1-E_1\bar\theta^2,\ \sigma_1^*=1-\sigma_{11}\bar\theta.\tag{4}$$

The expressions for the strains ϵ_r^*, ϵ_θ^* can be obtained in the following forms similarly as in [3].

$$\begin{aligned}
E^*\epsilon_\theta^* &= \frac{1-2\nu}{2(1-\nu)}\int\epsilon_\theta^*\frac{\partial E^*}{\partial\rho}\,d\rho+\frac{1}{2(1-\nu)}\frac{1}{\rho^2}\int\rho^2\epsilon_\theta^*\frac{\partial E^*}{\partial\rho}\,d\rho+(1+\nu)\frac{1}{\rho^2}\int E^*\left(\int\alpha^*\,d\bar\theta\right)\rho\,d\rho\\
&+\frac{1-2\nu}{2(1-\nu)}\int\frac{E^*}{\rho}[(\epsilon_r^{*p}+\epsilon_r^{*c})-(\epsilon_\theta^{*p}+\epsilon_\theta^{*c})]\,d\rho+\frac{1-2\nu}{2(1-\nu)}\frac{1}{\rho^2}\int\rho E^*[(\epsilon_r^{*p}+\epsilon_r^{*c})+(\epsilon_\theta^{*p}+\epsilon_\theta^{*c})]\,d\rho\\
&-\frac{\nu}{1-\nu}\frac{1}{\rho^2}\int\rho E^*\epsilon_z^*\,d\rho+B_0+\frac{B_1}{\rho^2},\\
E^*\epsilon_r^* &= \frac{1-2\nu}{1-\nu}\int\epsilon_\theta^*\frac{\partial E^*}{\partial\rho}\,d\rho-E^*\epsilon_\theta^*+(1+\nu)E^*\int\alpha^*\,d\bar\theta+\frac{1-2\nu}{1-\nu}E^*(\epsilon_r^{*p}+\epsilon_r^{*c})\\
&+\frac{1-2\nu}{1-\nu}\int\frac{E^*}{\rho}[(\epsilon_r^{*p}+\epsilon_r^{*c})-(\epsilon_\theta^{*p}+\epsilon_\theta^{*c})]\,d\rho-\frac{\nu}{1-\nu}E^*\epsilon_z^*+2B_0
\end{aligned}\tag{5}$$

where, $\rho=r/a$. The constants B_0, B_1 and ϵ_z^* can be obtained under the condition that the thick-walled tube is free from external forces[3].

$$\begin{aligned}
B_0 &= (1-2\nu)B_1,\\
B_1 &= \frac{1}{2(1-\nu)}\frac{R^2}{R^2-1}\left[2(1-\nu)\frac{1}{R^2}\int_1^R E^*\left(\int\alpha^*\,d\bar\theta\right)\rho\,d\rho\right.\\
&\qquad+\frac{1}{R^2}\int_1^R\rho E^*\{(\epsilon_r^{*p}+\epsilon_r^{*c})+(\epsilon_\theta^{*p}+\epsilon_\theta^{*c})\}\,d\rho\\
&\left.-\int_1^R\epsilon_\theta^*\frac{\partial E^*}{\partial\rho}\,d\rho+\frac{1}{R^2}\int_1^R\rho^2\epsilon_\theta^*\frac{\partial E^*}{\partial\rho}\,d\rho-\int_1^R\frac{E^*}{\rho}\{(\epsilon_r^{*p}+\epsilon_r^{*c})-(\epsilon_\theta^{*p}+\epsilon_\theta^{*c})\}\,d\rho\right],\\
\epsilon_z^* &= \frac{1}{\int E^*\,d\rho}\left[(1-\nu)\int_1^R E^*\left(\int\alpha^*\,d\bar\theta\right)\rho\,d\rho-\int_1^R\rho E^*\{(\epsilon_r^{*p}+\epsilon_r^{*c})+(\epsilon_\theta^{*p}+\epsilon_\theta^{*c})\}\,d\rho\right]
\end{aligned}\tag{6}$$

where, $R=a/b$. Expressions for the total strains ϵ_r^*, ϵ_θ^* have been derived in terms of the temperature distribution $\bar\theta$, and in terms of the total accumulated plastic strains ϵ_r^{*p}, ϵ_θ^{*p} and creep strains ϵ_r^{*c}, ϵ_θ^{*c}. Then the stresses σ_r^*, σ_θ^* and σ_z^* can be obtained from eqns (2).

2.2 *Temperature distribution*

The solution of the heat conduction equation for the thick-walled tube with zero initial temperature, where the temperature at the inner surface of the tube is maintained constant at T_0 for $t > 0$, is shown in [7] to be

$$\bar{\theta} = \frac{1}{K_1}[1 - (1 - 2K_1\psi)^{1/2}],$$

$$\psi = \psi_0\left(1 - \frac{\ln\rho}{\ln R}\right) + \pi\psi_0 \sum_{n=1}^{\infty} \exp(-\alpha_n{}^2 s)\frac{J_0(R\alpha_n)J_0(\alpha_n)U_0(\rho\alpha_n)}{J_0(\alpha_n) - J_0(R\alpha_n)} \tag{7}$$

where, $\psi_0 = 1 - K_1/2$. Provided that $J_0(x)$ and $Y_0(x)$ are the Bessel functions of the first and second kinds of order zero, $U_0(\rho\alpha_n)$ takes the form

$$U_0(\rho\alpha_n) = J_0(\rho\alpha_n)Y_0(R\alpha_n) - Y_0(\rho\alpha_n)J_0(R\alpha_n) \tag{8}$$

The α_n are positive roots of $U_0(\alpha_n) = 0$. The s in eqn (7) is the nondimensional time,

$$s = (h/a^2)t \tag{9}$$

where h means the factor of the diffusivity $h = K/\gamma C$, and may be taken as constant[7]. From eqns (4) and (7), we have

$$\frac{\partial E^*}{\partial\rho} = 2\bar{\theta}E_1(1 - 2K_1\psi)^{-1/2}\cdot\psi_0\left[\frac{1}{\rho\ln R} + \pi\sum_{n=1}^{\infty}\alpha_n\exp(-\alpha_n{}^2 s)\frac{J_0(R\alpha_n)J_0(\alpha_n)}{J_0^2(\alpha_n) - J_0^2(R\alpha_n)}\times\right.$$

$$\left.\{J_1(\rho\alpha_n)Y_0(R\alpha_n) - Y_1(\rho\alpha_n)J_0(R\alpha_n)\}\right]. \tag{10}$$

2.3 *Total accumulated plastic and creep strains*

The total accumulated plastic and creep strains can be obtained by summation of the increments of plastic and creep strains that occur during small intervals of time, each time interval corresponding to a particular change in temperature. Let the plastic strain increments $\Delta\epsilon_{ij}^{*p}$ and the creep strain increments $\Delta\epsilon_{ij}^{*c}$ be produced in the small interval Δs. Then total strains could be assumed to be

$$\epsilon_{ij}^* = \epsilon_{ij}^{*e} + \epsilon_{ij}^{*p} + \epsilon_{ij}^{*c} + \delta_{ij}(1-\nu)\int\alpha^*\,\mathrm{d}\bar{\theta} + \Delta\epsilon_{ij}^{*p} + \Delta\epsilon_{ij}^{*c}. \tag{11}$$

The time-independent plastic strain increments $\Delta\epsilon_{ij}^{*p}$ and the time-dependent creep strain increments $\Delta\epsilon_{ij}^{*c}$ in above eqn (11) can be obtained by the use of the constitutive equations which are independent of each other. This procedure is not correct in phenomelogical sense as the experiments show that these two components are not really different[8]. However, it leads to the results available to the actual engineering problems.

"Modified total strains" are now defined[6]

$$\epsilon_{ij}^{*\prime} = \epsilon_{ij}^* - (\epsilon_{ij}^{*p} + \epsilon_{ij}^{*c} + \Delta\epsilon_{ij}^{*c}). \tag{12}$$

The creep strain increments $\Delta\epsilon_{ij}^{*c}$ in above eqn (12), produced in the small interval Δs, are obtained independently from eqn (25) presented later. The deviatoric components are

$$e_{ij}^{*\prime} = \epsilon_{ij}^{*\prime} - \frac{1}{3}\epsilon_{ii}^{*\prime} = e_{ij}^{*e} + \Delta\epsilon_{ij}^{*p}. \tag{13}$$

With nondimensional deviatoric stresses s_{ij}^* $(= (1-\nu)s_{ij}/E_0\alpha_0 T_0)$, e_{ij}^{*e} is given by Hooke's law

$$e_{ij}^{*e} = \frac{1}{2G^*}\cdot s_{ij}^* \tag{14}$$

and from the Prandtl–Reuss equations

$$\Delta\epsilon_{ij}^{*p}/s_{ij} = \Delta\lambda. \tag{15}$$

Hence, from eqns (13) to (15)

$$e_{ij}^{*\prime} = \left(1 + \frac{1}{2G^*}\cdot\frac{1}{\Delta\lambda}\right)\Delta\epsilon_{ij}^{*p}. \tag{16}$$

If an "equivalent modified total strain", ϵ_{et}^* is defined as

$$\epsilon_{et}^* = \left[\left(\frac{2}{3}e_{ij}^{*\prime}e_{ij}^{*\prime}\right)\right]^{1/2} \tag{17}$$

then

$$\epsilon_{et}^* = \left(1 + \frac{1}{2G^*\Delta\lambda}\right)\left[\left(\frac{2}{3}\Delta\epsilon_{ij}^{*p}\Delta\epsilon_{ij}^{*p}\right)\right]^{1/2} = \left(1 + \frac{1}{2G^*\Delta\lambda}\right)\Delta\epsilon_p^*. \tag{18}$$

Since, by definition,

$$\Delta\epsilon_p^* = \left[\left(\frac{2}{3}\epsilon_{ij}^{*p}\epsilon_{ij}^{*p}\right)\right]^{1/2} \tag{19}$$

From eqns (16) and (18),

$$\Delta\epsilon_{ij}^{*p} = \frac{\Delta\epsilon_p^*}{\epsilon_{et}^*}\cdot e_{ij}^{*\prime}. \tag{20}$$

Equation (20) represents the Prandtl–Reuss equations, with the plastic strain increments expressed simply in terms of strain. With the equivalent stress $\bar{\sigma}^*$,

$$\bar{\sigma}^* = \left[\left(\frac{3}{2}s_{ij}^*s_{ij}^*\right)\right]^{1/2} \tag{21}$$

the relation $\Delta\lambda = (3/2)\cdot(\Delta\epsilon_p^*\sqrt{}\sigma^*)$ can be obtained from eqns (15), (19) and (21).
Then, from eqn (18),

$$1 + \frac{1}{3G^*}\cdot\frac{\bar{\sigma}^*}{\Delta\epsilon_p^*} = \frac{\epsilon_{et}^*}{\Delta\epsilon_p^*}. \tag{22}$$

Since $\bar{\sigma}^*$ depends on $\Delta\bar{\theta}$ and $\Delta\epsilon_p^*$, $\bar{\sigma}^*$ can be expressed as follows using Taylor's expansion,

$$\bar{\sigma}^* = \bar{\sigma}_{,i-1}^* + \left\{\left(\frac{\partial\bar{\sigma}^*}{\partial\epsilon_p^*}\right)_{,i-1}\Delta\epsilon_p^* + \left(\frac{\partial\bar{\sigma}^*}{\partial\bar{\theta}}\right)_{,i-1}\Delta\bar{\theta}\right\} + \dots \tag{23}$$

where, e.g. $\bar{\sigma}_{,i-1}^*$ represents the effective stress prior to the present increment of load, $\bar{\sigma}^*$ being the current value. From eqns (22) and (23) the expression for $\Delta\epsilon_p^*$ becomes

$$\Delta\epsilon_p^* = \frac{\epsilon_{et}^* - \frac{2}{3}\{(1+\nu)/E^*\}[\bar{\sigma}^*_{,i-1} + (\partial\bar{\sigma}^*/\partial\bar{\theta})_{,i-1}\cdot\Delta\bar{\theta}]}{1 + \frac{2}{3}\{(1+\nu)/E^*\}(\partial\bar{\sigma}^*/\partial\epsilon_p^*)_{,i-1}} \tag{24}$$

where higher order terms in $\Delta\epsilon_p^*$ and $\Delta\bar{\theta}$ are neglected.

The creep strain increments can be presented as follows with the Mises–Mises type theory of creep,

$$\Delta\epsilon_{ij}^{*c} = \frac{3}{2}\cdot\frac{\Delta\epsilon_c^*}{\bar{\sigma}^*}\cdot s_{ij}^* \tag{25}$$

where $\Delta\epsilon_c^*$ is the equivalent creep strain increment defined

$$\Delta\epsilon_c^* = \left[\frac{2}{3}\Delta\epsilon_{ij}^{*c}\Delta\epsilon_{ij}^{*c}\right]^{1/2}.$$

Neglecting the physical primary creep, the creep phenomena is presented by the following Norton's law[5].

$$\Delta\epsilon_c^* = A^*\bar{\sigma}^{*n}\cdot\Delta s,\ A^* = \frac{1}{A}\cdot\frac{E_0}{\sigma_{10}}\cdot\lambda^{n-1}\left(\frac{\sigma_{10}}{\sigma_n}\right)^n\cdot\left(\frac{a^2}{h}\right) \tag{26}$$

where A is constant with the dimension of time, σ_n and n are temperature-dependent material constants, and moreover, λ is called the loading parameter defined $\lambda = E_0\alpha_0 T_0/(1-\nu)\sigma_{10}$ as in [2 and 3].

2.4 *Procedure of numerical calculation*

To obtain the plastic strains, the following Ramberg–Osgood type stress–strain relation are used with the uniaxial stress σ_t and strain ϵ_t [4],

$$E^*e_t = s_t\left\{1+\frac{3}{7}\left(\frac{s_t}{\sigma_1^*}\right)^{m-1}\right\} \tag{27}$$

where $e_t = E_0\epsilon_t/\sigma_{10}$, $s_t = \sigma_t/\sigma_{10}$. From the above equation the relation between the equivalent stress and the plastic strain are as follows.

$$\bar{\sigma}^* = \left\{\frac{7}{3}E^*\epsilon_p^*\left(\frac{\sigma_1^*}{\lambda}\right)^{m-1}\right\}^{1/m}. \tag{28}$$

Therefore, the following term in the right hand side of eqn (24) becomes

$$\left[\bar{\sigma}^*_{,i-1}+\left(\frac{\partial\bar{\sigma}^*}{\partial\bar{\theta}}\right)_{,i-1}\cdot\Delta\bar{\theta}\right] = \bar{\sigma}^*_{,i-1}\left[1+\frac{\Delta\bar{\theta}}{m}\left\{\frac{(E^*)'}{E^*}+(m-1)\frac{(\sigma_1^*)'}{\sigma_1^*}\right\}\right]_{,i-1} \tag{29}$$

where $(\ )'$ means $\partial/\partial\bar{\theta}$. Equation (29) shows the variation of $\bar{\sigma}^*$ with temperature.

In the numerical calculations, the thick-walled tube is divided into 80 radial increments. The period during which the thermal load is applied is divided into 50 time increments. The computation of the plastic and creep strain increments that occur during a particular time increment (Δs) is carried out following an iterative procedure.

Computation begins with determination of the temperature distribution at time ($s+\Delta s$) with eqns (7) and (8). An iterative procedure is used to determine the total strains at each radial station. From eqns (5), (6) and (10), a first approximation to the total strains is obtained by setting the plastic strain increments to zero and the total accumulated plastic and creep strains to the values of these up to the end of the $(i-1)$th interval at time s, i.e. $\epsilon_{ij}^{*p}{}_{,i-1}$ and $\epsilon_{ij}^{*c}{}_{,i-1}$. On that occasion, the values of ϵ_z^* on the right hand side of eqns (5) and (6) should be set at first to the values of these at time s. The new estimated values of ϵ_z^* are then used to obtain a better approximation. After convergence, the strain ϵ_r^* can be computed from the second equation of (5). The "equivalent modified total strain" ϵ_{et}^* are obtained from eqns (13) and (17). At each radial station the values of ϵ_{et}^* are compared with $(2/3)\{(1+\nu)/E^*\}[\bar{\sigma}^*_{,i-1}+(\partial\bar{\sigma}^*/\partial\bar{\theta})_{,i-1}\cdot\Delta\bar{\theta}]$

(see eqn 24). If

$$\epsilon^*_{et} \leq \frac{2}{3}\{(1+\nu)/E^*\}[\bar{\sigma}^*_{,i-1} + (\partial\bar{\sigma}^*/\partial\bar{\theta})_{,i-1}\cdot\Delta\bar{\theta}],$$

then that particular radial station is situated in an elastic region, and correspondingly, $\Delta\epsilon^*_p$ is set to zero. On the other hand, if

$$\epsilon^*_{et} > \frac{2}{3}\{(1+\nu)/E^*\}[\bar{\sigma}^*_{,i-1} + (\partial\bar{\sigma}^*/\partial\bar{\theta})_{,i-1}\cdot\Delta\bar{\theta}],$$

then the particular radial station is situated in a plastic zone, and the approximate value of the "equivalent plastic strain increment" $\Delta\epsilon^*_p$ is calculated from eqn (24). First approximations are then calculated for $\Delta\epsilon^{*p}_{ij}$ from the modified Prandtl–Reuss relation (20). These values are then used to obtain a better approximation for the total strains ϵ^*_r and ϵ^*_θ. This process should be repeated as many times as is necessary to obtain the desired degree of convergence. After convergence, the stresses σ^*_r, σ^*_θ and σ^*_z can be computed from eqn (2). The accumulated plastic strains at time $(s+\Delta s)$ are now updated so as to include the plastic strain increments which have been determined by this iterative procedure. That is,

$$\epsilon^{*p}_{ij,i} = \epsilon^{*p}_{ij,i-1} + \Delta\epsilon^{*p}_{ij,i}. \tag{30}$$

Then the first approximation for the equivalent creep strain increment $\Delta\epsilon^*_{c,i}$ is computed from eqn (26) using the equivalent stress $\bar{\sigma}^*_{,i}$ obtained from eqn (21). With the stresses $\sigma^*_{ii,i}$ and $\bar{\sigma}^*_{,i}$, the first approximation for the creep strain increments $\Delta\epsilon^{*c}_{ij,i}$ are obtained from eqn (25). Adding these values $\Delta\epsilon^{*c}_{ij,i}$ to the accumulated creep strains at $(i-1)$-stage $\epsilon^{*c}_{ij,i-1}$, we have the first approximation for $\epsilon^{*c}_{ij,i}$ at i-stage.

$$\epsilon^{*c}_{ij,i} = \epsilon^{*c}_{ij,i-1} + \Delta\epsilon^{*c}_{ij,i}. \tag{31}$$

Computing the total strains $\epsilon^*_{ij,i}$ from eqn (5) and (6), and then stresses $\sigma^*_{ij,i}$ from eqn (2), we can obtain the new equivalent stress at i-stage $\bar{\sigma}^*_{,i}$ from eqn (21). The corresponding value of $\Delta\epsilon^*_{c,i}$ to this $\bar{\sigma}^*_{,i}$ are obtained from eqn (26). This value of $\Delta\epsilon^*_{c,i}$ is then used to obtain a better approximation for the creep strain increments $\Delta\epsilon^{*c}_{ij,i}$. This process should be repeated as many times as is necessary to obtain the desired degree of convergence. After convergence, the stresses σ^*_r, σ^*_θ and σ^*_z can be determined from eqn (2) and the accumulated creep strains are now updated so as to include the creep strain increments which have been determined by the iterative procedure described above. Computation then proceeds for the next increment of time. After the steady state of temperature is reached, the period during which only the creep deformation proceeds is divided also into 50 time increments (Δs). The same iterative procedure mentioned above is used to determine the total strains at each radial station with the computed values of the creep strain increments that occur during a particular time increment (Δs).

3. RESULTS OF NUMERICAL CALCULATIONS

For the stress–strain curves with $m = 50$ in eqn (27), which corresponds to those of carbon steel (0.40 Mn, 0.25 Si, 0.12 C) in [5], we assume the following thermal properties from [5]:

$$\alpha_0 = 11.7\times10^{-6}\,\mathrm{K}^{-1},\ K_0 = 59.9\,\mathrm{W/mk},\ E_0 = 206\times10^9\,\mathrm{N/m^2},\ \sigma_{10} = 235.4\times10^6\,\mathrm{N/m^2} \tag{32}$$

$$\alpha_1 = 0.0973\lambda,\ K_1 = 0.0307\lambda,\ E_1 = 0.00223\lambda^2,\ \sigma_{11} = 0.0774\lambda. \tag{33}$$

Poisson's ratio ν is assumed to be unaffected by temperature [2, 3] and to be equal to 0.4. Then, using these numerical values, it is easily shown that $\lambda = E_0\alpha_0T_0/(1-\nu)\sigma_{10} = T_0/58.6$. Moreover, the material properties for creep are also assumed from [5].

$A = 10^7$ (hr),

$n = -0.0267T + 17.15 = -1.565\lambda\bar{\theta} + 17.15,$

$$\sigma_n = \frac{137.3\times10^6}{2^{(0.02T-8)}} = -\frac{137.3\times10^6}{2^{(1.172\lambda\theta-8))}}\,\mathrm{N/m^2}. \tag{34}$$

In eqns (32) to (34), the thermal and material properties except the coefficients of thermal expansion have been obtained by the interpolation of the data given in [5] for the carbon steel, and the remainder is taken from [1]. Moreover, termal diffusivity is taken $h = 9(\text{mm}^2/\text{sec})$ from [5]. With the value of reference temperature $T_0 = 550°\text{C}$, i.e. $\lambda = 9.386$ and the strain hardening exponent $m = 50$, all the results presented in this paper are calculated for the thick-walled tube with the inner radius $a = 3(\text{mm})$ and outer one $b = 6(\text{mm})$, then $s = (h/a^2)t = t$. Special attention should be paid to eqn (34), in which the material properties for creep are not applicable to the temperature lower than 400°C, when the creep strains are not produced.

Figure 1 shows the Ramberg–Osgood's stress strain relation (27) with variation of temperature $\bar{\theta}$. With increase of the values of temperature, the yield stress σ_1^* remarkably decreases. The effect of temperature dependence on σ_1^* can be recognized more distinctly than that on the elastic modulus E^*.

Figure 2 shows the variation of temperature $\bar{\theta}$ with respect to ρ for different values of time s. The numerals in the figure denote the values of elapsed time after rapid heating. The temperature distributions in the steady state are obtained at short time $s = 2$. The dash-dot line, i.e. $\bar{\theta} = 0.727$ indicates the lowest temperature at which the creep strains are produced. Even in steady state of temperature, the creep deformation can be recognized only in the narrow range from the inner surface to $r/a = 1.18$.

Figures 3 to 5 show the variation σ_r^*, σ_θ^* and σ_z^* with respect to ρ at $s = 0.01$, 0.1 and 2, and the relaxation of these stresses at $s = 10000$ under the steady state of temperature. The creep strains in the transient state of temperature are so small, as shown later in Fig. 7, that the stress distributions with and without consideration of these creep strains can not be distinguished in these figures. The radial stress σ_r^* which is clearly compressive drops a little at $s = 10000$. The circumferential and axial stresses σ_θ^* and σ_z^* are compressive from the inner surface to the central portion of the tube except the extremely narrow region including that surface where these stresses become to be tensile with time because of unloading after yielding, similarly as in [2], and tensile from that portion to the outer surface. The relaxation of both stresses σ_θ^* and σ_z^* are recognized clearly from $r/a = 1$ to 1.2. Since no creep strains are produced in the other range of the tube as mentioned above, the distributions of stresses at $s = 10000$ have no difference from those at the steady state of temperature $s = 2$ except that range, i.e. from $r/a = 1$ to 1.2.

Figures 6 and 7 show the variation of creep strain ϵ_z^{*c} in the transient and the steady state of temperature, respectively, with respect to s at the inner surface of the tube, i.e. $r/a = 1$. The creep strains in the transient state are about 1/100 of those in the stress relaxation, and therefore can be neglected without loss of accuracy in the numerical calculations.

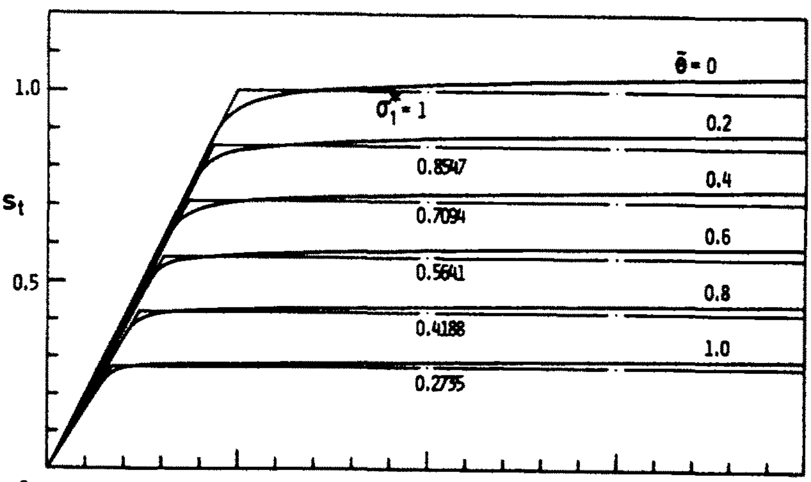

Fig. 1. Ramberg–Osgood's stress strain relation.

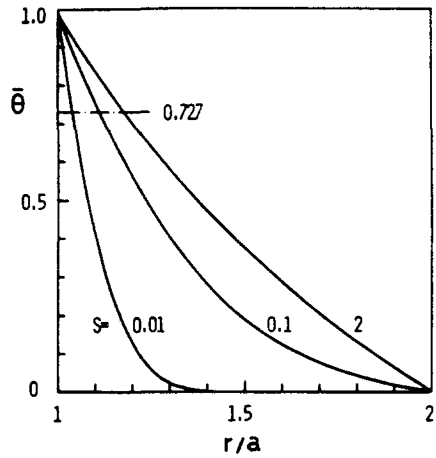

Fig. 2. Temperature distributions.

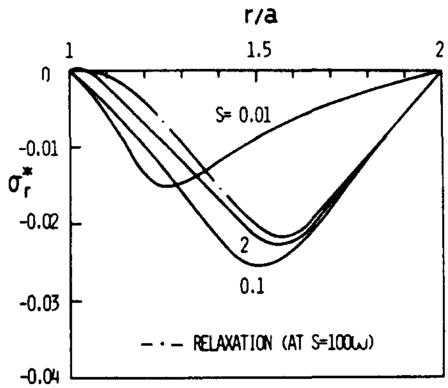

Fig. 3. Radial stress σ_r^*.

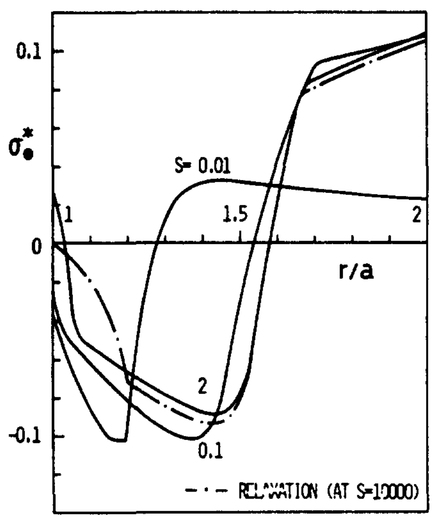

Fig. 4. Circumferential stress σ_θ^*.

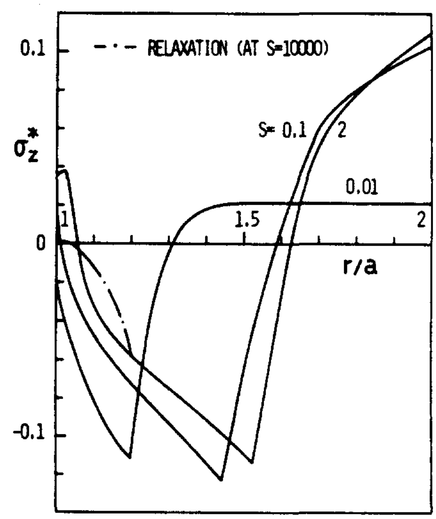

Fig. 5. Axial stress σ_z^*.

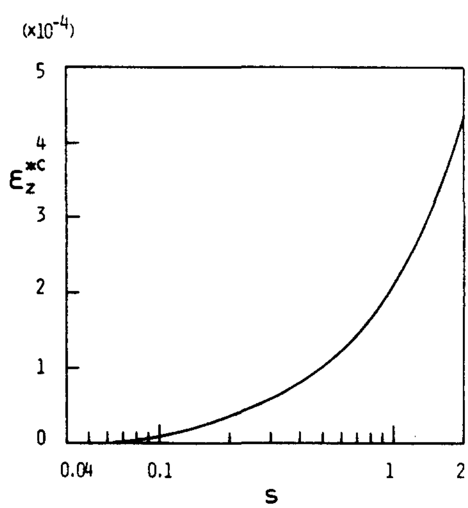

Fig. 6. Axial creep strain ϵ_z^{*c} in a transient state under heating.

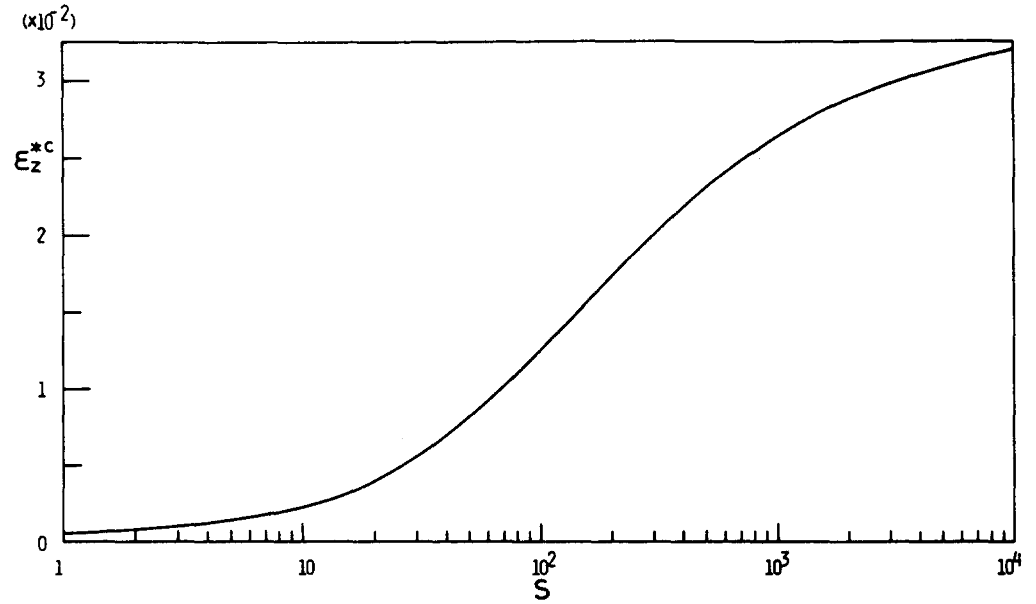

Fig. 7. Axial creep strain ϵ_z^{*c} in stress relaxation.

4. CONCLUSION

The thermoelastoplastic creep deformation during the transient state of temperature and the relaxation of thermal stresses at the steady state in the thick-walled tube after a sudden temperature rise on its inner surface are presented. The material of the tube is assumed to have temperature dependent properties and to be characterized by the Ramberg–Osgood's stress strain relation and Norton's law for secondary creep. An illustrative example of the carbon steel (0.40 Mn, 0.25 Si, 0.12 C) with the inner radius $a = 3$(mm) and the outer one $b = 6$(mm) shows the following conclusions:

(1) After being subjected to rapid heating $T_0 = 550°C$ at the inner surface of the tube, the temperature of the body becomes stationary in a short time, i.e. $t = 2$(sec). The creep strains during this period are small and can be neglected in the stress analysis.

(2) After the steady state of temperature, the creep strains becomes larger with time elapsed, and then the relaxation of stresses can be recognized.

(3) The relaxation of stresses σ_θ^* and σ_z^* are larger than that of σ_r^*, and concentrate in the vicinity of the inner surface of the tube where the temperature is higher than 400°C.

(4) This paper deals with the statical primary creep in a thick-walled tube subjected to a transient temperature distribution using Norton's law. The consideration of the physical primary creep seems to lead to larger creep strains during the transient state of temperature, but this should be discussed later in the other papers.

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